Quantum Fourier Transform w/ Leakage Ctrl

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Abstract:
In circuit QED (cQED) systems we can manipulate the cavity Fock states to use a quantum processing unit or QPU. Because of the long coherence times associated with Fock states, ”qudit” based computations are possible. The quantum Fourier transform QFT is instrumental in the synthesis of selective number-dependent arbitrary phase gate and displacement gates that comprise a QPU.

The QFT is the quantum mechanical version of the classical Fourier transform which maps signals in the continuous time domain into the frequency domain. The quantum Fourier transform maps signal into the phase domain. Posited over complex variables, the mathematics is simpler in phase space. We report conjectured results at the end.

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- space Cost functions with and without leakage control

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Acknowledgements

Phase space

QFT produces amplitudes that in phase space that can define a signal in a more mathematically convenient manner than frequency space. Frobenius norms and related calculations are simpler.

Complex conjugates abound and if these computations were taken over the reals, mathematical difficulties arise.

$$a, b \in \mathrm{R}, a \pm b\, i \in \mathrm{C} \text{ where } i \text{ is imaginary}$$

Cost function w/o leakage control

Our na¨ıve cost function:

$$\mathrm{L} = 1 - \left| \frac{1}{N_c} \mathrm{Tr}\ U(\alpha, \theta) U_T^{\dagger} \right| \tag{1}$$

where $N_c \times N_c$ U_T is the target gate to engineer, and $N_c \times N_c$ $U(\alpha, \vec{\theta})$ is the gate that we can engineer with SNAP and displacement blocks.

Cost function w leakage control

Our na¨ıve cost function:

Thus, we define a new cost function that penalizes the leakage after each displacement operator. We define the cost function as

$$L_p = 1 - \frac{\mathrm{Tr}\ U^{\dagger} U(\vec{\alpha}, \vec{\theta}) Q}{d} + \sum_{k=1}^{K} w_k \frac{|| P U_k(\vec{\alpha}, \vec{\theta}) Q ||^2}{d}, \quad (2)$$

where $||(A)|| = \sqrt{\mathrm{Tr}(A^{\dagger} A)}$ is the Frobenius norm, and w_k is the arbitrary weight we can define for each layer.

SNAP and displacement gates

To facilitate qudits in circuit quantum electrodynamics (cQED) systems we alter the state of these qudits via SNAP (Selective Number-dependent Arbitrary Phase) and displacement gates. The QFT is used to synthesize them in alternating configurations.

As stated, $U(\alpha, \vec{\theta})$ is the gate that we can engineer with SNAP and displacement blocks. The $U(\alpha, \vec{\theta})$ is defined as,

$$U(\alpha, \vec{\theta}) = \prod_{k=1}^{K} B(\alpha, \vec{\theta})_k \tag{3}$$

and the blocks are defined as

$$B(\alpha_k, \vec{\theta}_k) = D(\alpha_k) S(\vec{\theta}_k) D(-\alpha_k). \tag{4}$$

Bumper states

We can introduce projector operators

$$Q = \sum_{n=0}^{N-1} |n\rangle\langle n|, \quad (5)$$

which selects the logical states, and then Thus, we introduce another projector operator that selects the bumper states

$$P = \sum_{n=d}^{N_b-1} |n\rangle\langle n|, \quad (6)$$

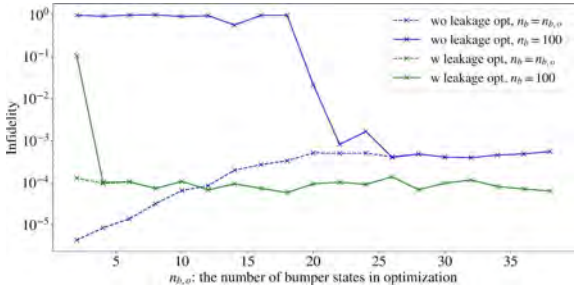
such that if we define the target unitary as

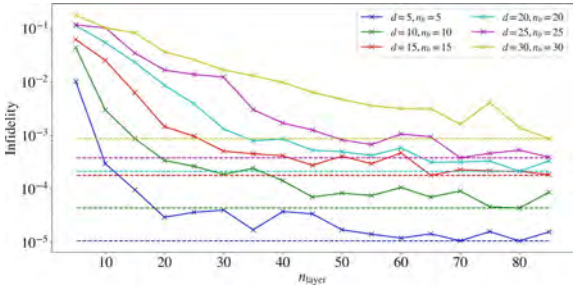
$$U_T = \begin{matrix} \mathrm{F} & 0 \\ 0 & \mathrm{B} \end{matrix}, \quad (7)$$

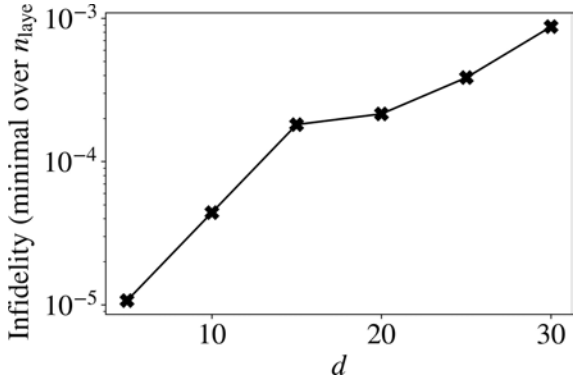
$$QU_TQ = \mathrm{F} \quad (8)$$

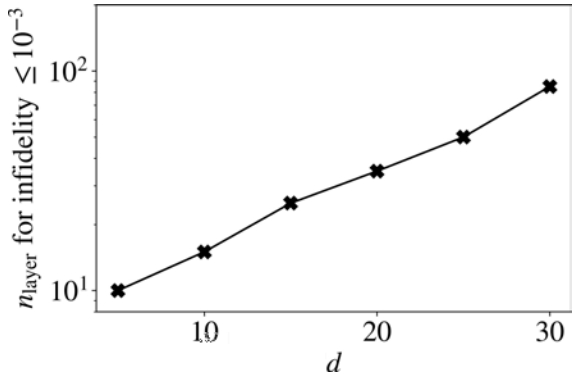
Some early tresults,

To be interpreted and refined. Further simulations are in progress.









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